GEOMETRIC AND AESTHETIC DISCRETIZATION OF FREE FORM SURFACES

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Resume

The development of digital technologies in the last twenty years has led to an unprecedented formal freedom in design and in the representation in virtual space. Combining non-standard geometry with CAD tools enables a new way of expression and realization of architectural ideas and conceptions. The transformation of a virtual double-curved surface into a buildable physical structure and object is always accompanied by huge costs and big problems like geometric and statical ones.
This paper shows geometric methods how to control the construction of curved surfaces out of planar building elements. The approach is based on the discretization of the surfaces by plane elements derived from tangent planes. In order to satisfy also aesthetical requirements we engage plane geometrical patterns and ornaments and transfer them into spatial shape.

Key words: Free-form surfaces, discretization, patterns, symmetry groups

1. INTRODUCTION

Unconventional geometric shapes and free-form surfaces - also known as non-standard geometry - have always been something that architects have wanted to design and build. In the history of architecture many of these forms could not be conceived as the design process was restricted by representation media and scale. The development of digital technologies in the last twenty years has led to an unprecedented formal freedom in design and in the representation in virtual space. New CAD software supports the generation and

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modification of various geometries like solid objects with extrusions, Boolean operations, transformations etc. Furthermore CAD made it possible to work with complex NURBS surfaces using splitting, lofting and sweeping techniques etc. This has opened up another large domain. Forms that are generated this way can be very complex, single or double curved or consist of polygonal faces.

Combining non-standard geometry with CAD tools enables a new way of expression and realization of architectural ideas and conceptions. Non-standard geometry has become a fixture in the work of many of the world's foremost star-architects. Given the high degree of attention these geometric extravaganzas garner it seems astonishing, almost paradoxical, that the field of architecture as a whole is not investigating such geometries more thoroughly. In this age of digital-virtual architecture where complex non-standard architectural forms are possible there really is only a small number of architects that have acquired the know-how to make use of this enormous potential which makes non-standard architecture more buildable.

But within the architectural design, engineering, fabrication and construction communities there is a growing interest in the potential of digital technologies and parametric design for a change of professional practice.

2. DISCRETISATION OF FREE-FORM SURFACES

The discretization of free-form surfaces has been a topic of great interest in the last few years. Discretisation is the first step in the creation of buildable free forms in architecture. Free-form surfaces may be discretized in a number of ways and in accordance with a number of principles.

One method of surface discretization results in curved segments (Figure 01), whereas with another planar panels are obtained. In terms of geometry, there are two fundamental differences between these two methods of discretization.

Figure 01 *Discretization with curved segments, Kunsthaus Graz*

First: with the first method of discretization, the set form is not approximated; instead, the complete form is segmented into smaller curved segments. When a surface is discretized into planar elements, the initial surface form is approximated in order to obtain planar elements, and depending on the size of individual elements, a greater or lesser distortion in the geometry of the set form occurs.

Second: when a surface is discretized into planar elements, the obtained segments of the surface share straight boundary edges, while with curved segments the boundary edges are curved.

Nevertheless, if a surface is to be discretized in view of its actual construction, obviously from the aspect of technology discretization into planar elements is both more feasible and cost-effective, in comparison with discretization into curved elements. From an aesthetic point of view, discretization into curved elements offers many more forms to choose from in shaping individual segments to choose from, while planar discretization offers a limited number of segment forms.

In our approach we will concentrate on discrete forms and surfaces, approximating complex curved shapes with flat panels which keeps costs down independent of the choice of buildings material.

3. PLANARIZATION

3.1 Theory

A free-form surface can be segmented into plane elements using different techniques as triangulation (Figure 02, left), quad meshing (Figure 02, right), or more or less freely placed tangent planes. There are both advantages and disadvantages to each of these methods of discretization, which will be discussed briefly.

Triangulation is the best-known method of curved surface discretization (Figure 03). This method is used for partitioning a selected surface into triangular planar segments. The drawback of this method is a very large number of edges with a high degree of geometric complexity, which in turn requires a big number of load-carrying members, great quantities of structural materials, and increases construction costs. When it comes to aesthetics, only the size and aspect ratio of triangular panels can be influenced.

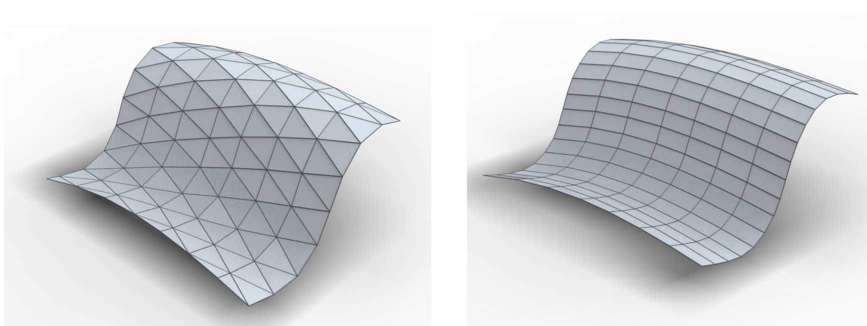

Figure 02 Regular triangulation (left) and quad meshing (right)

The second method is quad meshing, where a surface is divided into quadrangular polygons [Glymph et. al 2002, Pottmann et. al. 2008]. From the aspect of use of material, quad meshing is more optimal than tessellation into triangular elements. However, it cannot be employed exactly with arbitrary surfaces, but only with surfaces generated in a special way (e.g. extrusion, translation or rotation).

Figure 03 *Triangulation, Murinsel Graz*

The third type of plane discretisation of free-form surfaces is as follows. An arbitrary set of points is distributed on the surface and their tangent planes are constructed. The solution is based on the intersection of the tangent planes of the surface. The fact that there is an infinite number of possibilities when selecting points on a surface through which tangent planes can be placed raises the issue of the way and conditions which make it possible to select specific tangent planes whose intersection would produce the desired shape in accordance with the previously selected tessellation, a 3D ornament. Another issue is whether there is an infinite range of possibilities to generate a preferred 3D ornament and on what conditions surface tessellation would be ornamental in character, i.e. it would generate not only the functional, but also the aesthetic component of a free-form surface. The figures 04 – 07 show the different steps from the choice of ornament in the [uv]-parameter plane (figure 04) to the curved ornamental patterns (figure 05) over to their associated tangent planes before their intersections (figure 06) and finally after their intersections.

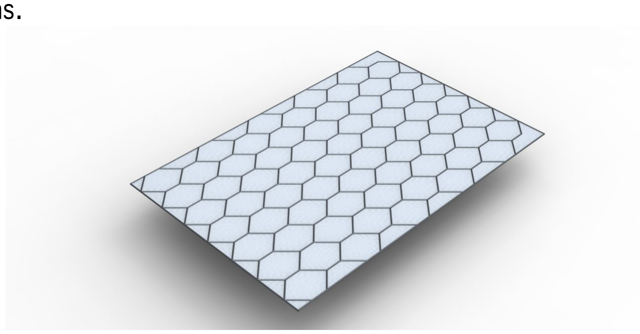

Figure 04 *2D ornament*

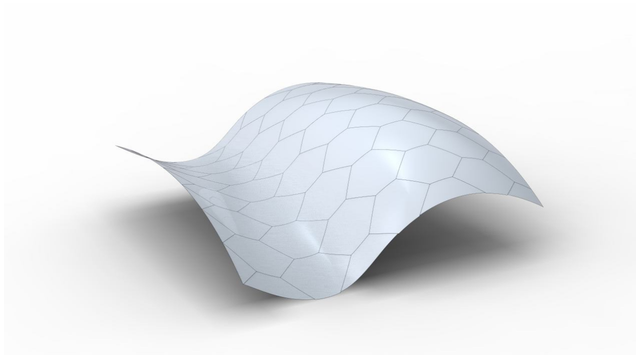

Figure 05 *3D ornament*

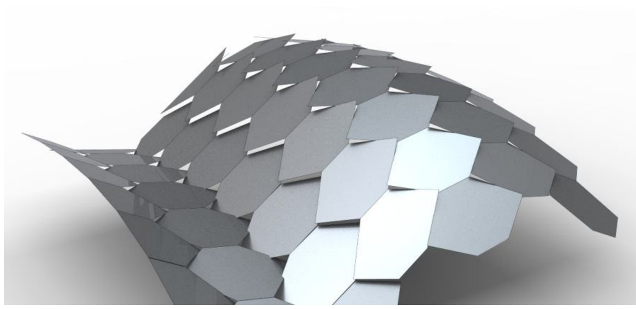

Figure 06 *3D plane ornaments*

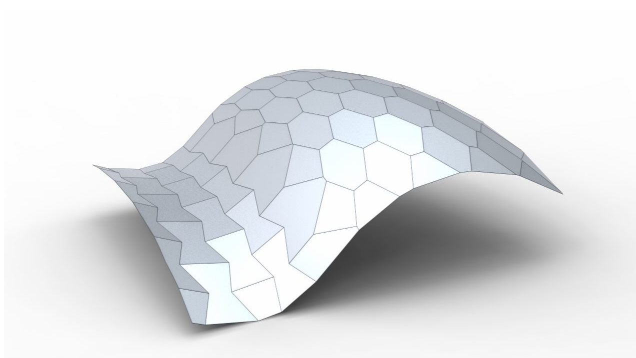

Figure 07 3D discretization

3.2 Implementation

In the process of discretizing a curved surface with planar segments, one constantly comes across intersecting planes. The edges created by intersecting planes define the boundary lines between panels and it is necessary to know how these edges behave in order to select those tangent planes which will result in the preferred intersection orientation.

The paper [Troche, 2008] shows the shape of the polygon greatly depends on the local curvature of the surface. The elliptical, parabolical and hyperbolical points of a surface will result in different types of concave or convex polygons. Therefore, it is necessary to analyze any given surface based on its elliptical and hyperbolical surface areas, dividing it into segments having by the same kind of surface points (Figure08). Surfaces which contain only parabolic points are not of our interest.

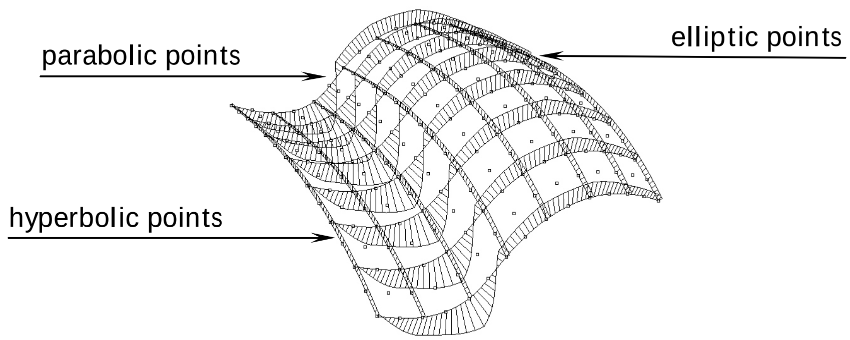

Figure 08 *Analyse of surface curvature along isocurves, parabolic, hyperbolic and elliptical points*

Our approach to the discretization of a (double-) curved surface S is as follows. We take a number of surface points P_i (i = 1, ..., n) which (at this stage) are arbitrarily distributed on S. To every point P on S we determine its tangent plane τ_P and intersect it with the tangent planes of adjacent points. We keep that polygonal part of the intersected plane τ_P which encloses P. This part is convex if P is an elliptical surface point and concave if P is hyperbolic. An important question at this stage is which points and tangent planes are adjacent to P in order to carry out correct intersections. One known approach is to use a Delaunay triangulation to figure out the nearest and neighbouring points. This can be done by a triangulation of the parameter value set (u_i, v_i) of the points P_i in the [u,v] – plane or as a spatial Delaunay triangulation of the points P_i. This triangulation is used to determine the first circle of adjacent points, which takes into account only the physical layout of points. This works only in special cases and causes in general often problems (see e.g. Troche, 2008). This is due to the fact that this approach neither considers an adapted surface measurement in form of geodesic lines nor takes the curvature of the involved surface in account.

In our approach that triangulation leads to the next stage of calculations, with the selection of second-order adjacency points in relation to the selected local points. These points will be used in the second stage of triangulation.

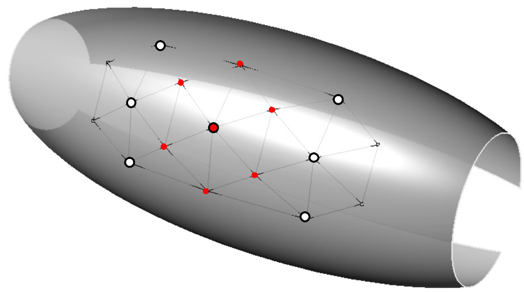

Figure 09 *Delaunay triangulation of some points on a curved surface*

We developed the following idea to construct an appropriate triangulation of our point set P_i on an arbitrary double curved surface and therefore to get a correct intersection algorithm for the tangent planes. For every point P on S we perform a transformation of S and our point set P_i so that for the mean curvatures κ_1 and κ_2 in P the following holds:

$$|\kappa_1| = |\kappa_2|$$

After that we project the neighbouring points of the point set P_i into the tangent plane of the point P. In the tangent plane we now perform the Delaunay triangulation. This yields a correct correlation between the surrounding points and so we can intersect the involved tangent planes to generate the polygonal panel associated with P.

4. EXAMPLES

The following figures (Figure 10, Figure 12) show some examples where the point set on the surface was varied by moving them along uv value(Figure11). So we got various patterns although we didn't change the input surface nor the number of points. We only changed the position of several points on the surface and therefore their tangent planes. This led to different intersections and so to different panels.

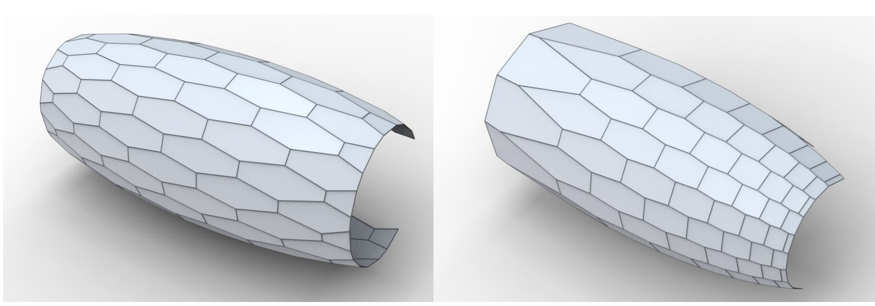

Figure 10 *Different panels form*

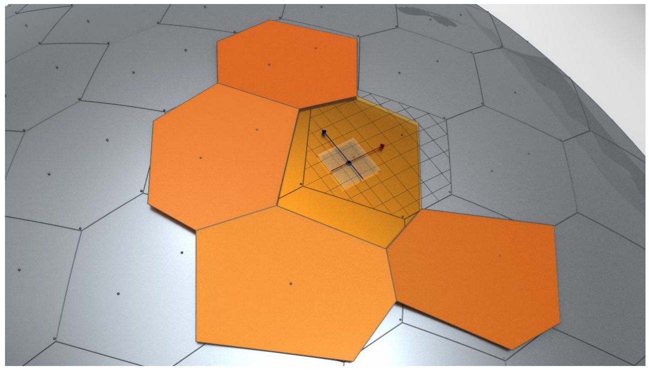

Figure 11 *Different uv-values and their influence on the shape*

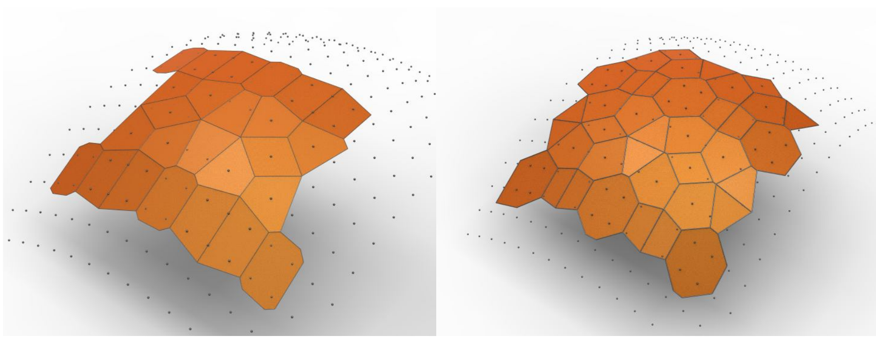

Figure 12 *Different panels form*

5. ACKNOWLEDGEMENT

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6. LITERATURE

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